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An Uncorrelated Variable Step-size LMS Adaptive Algorithm

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ABSTRACT

In this paper, we discuss and analyze two variable step-size adaptive algorithms in which the performance is reduced when input signal are highly correlated with each other. As a result, we propose an uncorrelated variable step-size LMS adaptive algorithm in order to overcome that shortcoming. The main idea of this algorithm is to pretreat input signal by decorrelation then use this pretreated signal to update weight vector of the adaptive filter. Due to the theory of correlation, the algorithm can achieve good convergence rate. Finally, the simulations are consistent with the theoretical analysis.

Keywords: *LMS; variable step-size; uncorrelated*

1. INTRODUCTION

Adaptive gradient algorithm includes LMS algorithm and various improved LMS-type algorithms [1]. In 1960, Widrow presented Least Mean Square (LMS) algorithm which could be widely used in the area of automatic control, radar, signal processing, etc. because it is simple both in realization and calculation. But the fixed step-size LMS algorithm has contradictory requirement in convergence rate and Steady-state error. In order to solve this problem, people have developed many variable step-size LMS algorithms. VS-LMS algorithm [2] is one kind of algorithm using variable step-size instead of fixed step-size. However, MS-LMS algorithm [3] is improved based on VS-LMS algorithm, and it achieves better performance both in convergence rate and Steady-state error. These two kinds of algorithms are proposed based on input signals are independent of each other in any two time points. In fact, the slowdown of its convergence rate will lead to the reduction of performance when input signals are highly correlated with each other. So we can use the theory of correlation to deal with the correlation of input vector.

2. ALGORITHM ANALYSIS

LMS algorithm is given by the following iteration equation:

$$y(n) = W^T(n)X(n);$$
$$e(n) = d(n) - y(n);$$
$$W(n+1) = W(n) + 2\mu e(n)X(n).$$

where $y(n)$ is the output of adaptive filter, $W(n)$ is the weight coefficient vector of adaptive filter, $X(n)$ is the input vector, $d(n)$ is the desired output, $e(n)$ is the error signal, μ is the step-size.LMS algorithm can converge when $0 < \mu < 1/\lambda$, in which λ is the maximum eigenvalue of input signals' autocorrelation matrix.

When we increase the step-size, the adaptive filter's convergence rate will be faster. At the same time, steady-state error will increase. In order to solve this problem, people developed many variable step-size LMS algorithms. Gao et. al[2] analyzed many variable step-size adaptive filter algorithms, then propose a new variable step-size LMS algorithm. We call it VS-LMS algorithm which showed by the following iteration equation:

$$y(n) = W^T(n)X(n);$$
$$e(n) = d(n) - y(n);$$
$$\mu(n) = \beta(1 - \exp(-\alpha|e(n)|^2));$$
$$W(n+1) = W(n) + 2\mu(n)e(n)X(n).$$

This algorithm can achieve both fast convergence and small steady-state error. Gao et. al [2] not only analyzed the method to set up parameters α and β, but also analyzed the parameters' influence to the algorithm's performance. Xu et. al[3] analyzes the shortcoming of VS-LMS in low signal-to-noise ratio environment. Then, Xu et. al[3] uses error correlation value $|e(n)e(n-1)|$ to instead of error squre $|e(n)|^2$to regulation step-size. This new variable step-size LMS algorithm we call it MS-LMS which retains the VS-LMS algorithm's advantage and

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achieves more excellent performance in low SNR environment. So MS-LMS can be used more widely. MS-LMS algorithm is given by the following iteration equation:

$$y(n) = W^T(n)X(n);$$
$$e(n) = d(n) - y(n);$$
$$\mu(n) = \beta(1 - \exp(-\alpha|e(n)e(n-1)|));$$
$$W(n+1) = W(n) + 2\mu(n)e(n)X(n).$$

These two LMS algorithms are better than fix step-size LMS algorithm, but they are proposed based on the principle that input signals are independent with each other in any two time points. When input signals are highly correlated with each other, algorithm performance will be affected, especially in reducing convergence rate. In this case, we can remove the correlation to make the input vector independent as far as possible. Research shows that decorrelation can effectively accelerate the convergence rate [1].

3. AN UNCORRELATED VARIABLE STEP-SIZE LMS ALGORITHM

Doherty et. al[4] advances if we make a decorrelation treatment for input signal before it be put into the adaptive filter, we can improve algorithm's performance. It defines a correlation coefficient similar a projection coefficient in LMS algorithm: $a(n) = \frac{X(n)^T X(n-1)}{X(n-1)^T X(n-1)}$

Where $a(n)$ is the correlation coefficient between $X(n)$ and $X(n-1)$ at time n. The $a(n)$ is larger the correlation is stronger. Obviously, $a(n)X(n-1)$represent the correlation part between $X(n)$ and $X(n-1)$.We deduct this correlation part from $X(n)$.This subtraction equivalent to "decorrelation". Then, we use $Z(n)$ the result of correlation to update weight vector.

$$Z(n) = X(n) - a(n)X(n-1).$$

Combine MS-LMS algorithm with decorrelation pretreatment, we can propose an uncorrelated variable step-size LMS algorithm (DMS-LMS). It can be showed by the following iteration equation:

$$y(n) = W^T(n)X(n);$$
$$e(n) = d(n) - y(n);$$
$$a(n) = \frac{X(n)^T X(n-1)}{X(n-1)^T X(n-1)};$$
$$Z(n) = X(n) - a(n)X(n-1);$$
$$\mu(n) = \beta(1 - \exp(-\alpha|e(n)e(n-1)|));$$
$$W(n+1) = W(n) + 2\mu(n)e(n)Z(n).$$

Refer to Gao et. al[2] to set up parameters α and β.

4. THE COMPUTER SIMULATION AND ANALYSIS

In this section, we describe simulations performed to verify the theory developed in the previous section, and compare experimentally the performance of the new DMS-LMS algorithm to that of the MS-LMS algorithm.

The simulation model is shown in figure 1 and simulation condition is setup as followed:

1) 5-tap adaptive filter;
2) The impulse response of unknown system is: h=[0.8;0.5;0.3;0.2;0.1];
3) The white Gaussian noise channel, in which SNR equal to 20 dB and 10 dB respectively;
4) Input signal is x(n)=0.9x(n-1)+N(n), where N(n) is a white Gaussian noise signal for variance is 0.1 and zero mean;
5) α=10，β=0.1;
6) Number of samples is 2000.

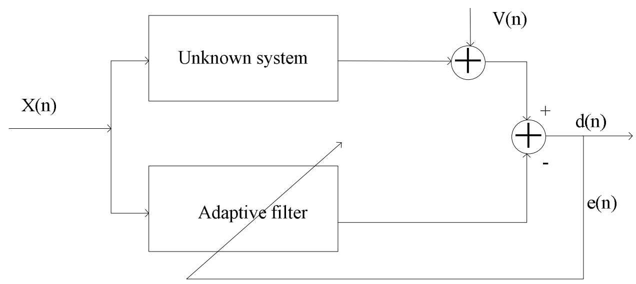

Fig. 1: Adaptive filter diagram

We simulate 100 times independently to draw a convergence curve by statistical average as is shown in figure 2.

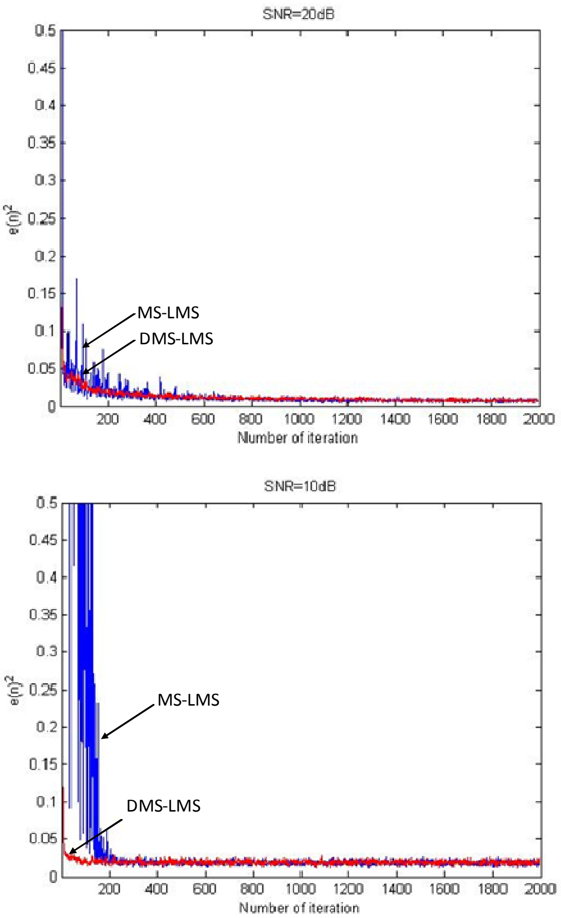

Fig.2 Convergence curve

From figure 2, we can see when input signals are highly correlated with each other; convergence rate of DMS-LMS is faster than it of MS-LMS. DMS-LMS algorithm can keep convergence rate both in the case of SNR is 20 dB and 10 dB. In deepness convergence, Steady-state performances of DMS-LMS and MS-LMS are exactly similar. So we can conclude when input signals are highly correlated with each other, decorrelatoin algorithm can increase convergence rate in the adaptive process.

5. CONCLUSIONS

An uncorrelated variable step-size LMS algorithm has been introduced which uses the step-size equation in MS-LMS to update step-size and pretreats input signal by decorrelation then uses this pretreated signal to update weight vector of the adaptive filter. Comparison shows when input signals are highly correlated with each other; convergence rate of DMS-LMS is faster than it of MS-LMS. The simulations are consistent with the theoretical analysis.

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