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Networks of Aligned Ontologies Through a Category Theoretic Approach

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ABSTRACT

A network of ontologies is a distributed system, whose components (constituent ontologies) are interacting and interoperating, the result of this interaction being, either the extension of local assertions, valid within each individual ontology, to global assertions holding between remote ontology entities, or to remote induced local assertions holding between local entities of an ontology, but induced by remote ontologies. The mechanism for this interaction is entailment and a crucial issue is to characterize the global, network induced entailments and propagate local knowledge through the network, in order to retain the consistency of the whole network and, thus, to extract meaningful results from the global knowledge emerged from a particular network of aligned ontologies. In this paper, we focus only on the first step of this research direction, that is, the ontology representation as a path category upon which operations will be defined in the future, in order to formalize the characterization of entailments in a network of aligned ontologies.

Keywords: *Category Theory, Ontologies, Ontology alignment*

1. INTRODUCTION

Category Theory (CT) is a branch of pure mathematics that becomes an increasingly important tool in theoretical computer science [3, 13, 25], especially in knowledge representation and management, by means of ontologies and ontology alignments [21].

Ontologies are formal knowledge representations of a domain, described by concepts that are types, or classes of things, objects that are instantiations of classes, attributes that are object features and relations between concepts and objects [16]. An ontology is a knowledge source expressed in its own vocabulary and which also provides axioms governing the different terms in this vocabulary and thus constraining the possible interpretations of the language. Two different ontologies – knowledge sources, could however use different knowledge representation languages (syntactic heterogeneity), could also use different vocabularies (terminological heterogeneity), or could have a different modeling of a common vocabulary (semantic heterogeneity) [32]. These sources of heterogeneity should not prevent the possibility of the interpretation of knowledge.

Since, in order to avoid heterogeneity, a unique ontology and a unique knowledge representation language are unrealistic to be imposed, relations are established between the ontologies representing different knowledge sources. The process of finding relations holding between different ontologies is called ontology matching and the result of ontology matching, i.e. the declarative expression of the relations holding, is called an alignment between the ontologies.

Some approaches followed in the literature for the formalization of ontologies and their operations; consist of using the Information Flow Theory [4], the Formal Concept Analysis (FCA) [35] and the Goguen's work on Institution Theory. The basic idea behind Information Flow (IF) Theory, is the notion of containment, which is interpreted as “the information an object contains about another”. The IF Theory has been used in ontology operations, since through the notions of infomorphism and channel enable semantic interoperability at the type and the token level [1, 18, 20, 22]. FCA has been used as means for modeling and analyzing semantic information [15, 27, 36], since it is a method for analyzing data which describes relationships between a set of objects and a set of attributes. From the input data, FCA produces a concept lattice, which is a collection of formal concepts in the data. Formal concepts are clusters representing natural human-like concepts and are hierarchically ordered by a subconcept-superconcept relation. FCA also produces attribute implications which describe particular dependencies valid in the data. Institutions, as an adequate category-theoretic tool, have been used to address the semantic heterogeneity problems [23]. A limitation of these approaches is that it is assumed a shared understanding of semantics.

In case, where comparison between ontologies is needed at multiple levels, namely at the entity level, at the ontology level and finally at the network of interlinked ontologies level, Category theory is suggested as the most appropriate formalization framework, since it:

- Focuses on relationships (categorical morphisms, functors, and natural transformations) and not on entities (categorical objects, and categories),
- Allows the coexistence of heterogeneous entities, since it provides the ability to define several categories, according to the kinds of entities to be described (category of ontologies, category of alignments, category of networks of interlinked ontologies), which can be related by the definition of special morphisms (categorical functors),
- Offers a set of categorical constructors for creating new categories, by using predefined ones,

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- Provides a means for the combination of categorical objects (co limits which are used to compose them, limits to decompose them), and for the combination of categorical functors (natural transformations),
- Provides a multi-level study of its categorical notions, by defining three interrelated levels (the level of categories, the level of functors and the level of natural transformations).

The difficulty in this multi-level nature of studying categorical notions, is to find out the appropriate way to get from one level to the other. This can be achieved, if every categorical notion has been explicitly specified and defined in all the three different levels. This is essential because a notion is simplified when we examine it from an upper level.

In a recent work [30] we used Category Theory in order to embed the Yoneda philosophy in conjunction with Formal Concept Analysis, into ontology engineering research issues, such as the conceptualization phase of each ontology building methodology.

The emphasis in this paper is on offering suggestions for future research directions in using more elaborate categorical ideas, such as the Yoneda embedding functor and enriched categories, in order to characterize logical consequences in networks of aligned ontologies.

The remainder of the paper is organized as follows. The problem to be solved is represented in Section 2. Categorical constructions that are already used heavily in the formalizing ontologies and their operations, such as merging, alignment and mapping are discussed in Section 3. Section 4 reports on some recent theoretical work in networks of ontologies linked by alignments. Section 5 provides the categorical ideas, such as the path category associated to an ontology, which will be extended in future work, in order to construct an initial formulation for propagating relations in a network of ontologies and alignments and thus characterizing logical consequences. Section 6 concludes the paper, while in Section 7, future directions, such as, the definition of composition operators permitting to combine knowledge along chains of ontologies and alignments in a network, are discussed.

2. NETWORK OF ALIGNED ONTOLOGIES

Applications in open, dynamic and distributed environments, such as pervasive computing environments and the Semantic Web need to share resources. These applications involve autonomous entities which have been designed independently [11, 26, 28]. Thus, they are facing a high level of heterogeneity. On the one hand this is desirable, as it allows the involved parties to structure knowledge in a way fitting their needs best, e.g., regarding a specific application. On the other hand, this is problematic, as it impedes the involved parties' communication because knowledge of the resources is encoded in a variety of ways. One aspect of overcoming heterogeneity in order for the involved entities to interoperate in these environments is an explicit and semantically rich representation of knowledge through ontologies. Ontologies aim at capturing domain knowledge in an explicit way and they provide a consensual understanding of the domain [6, 7]. Because it is impractical for all the involved entities to share a unique and global ontology, a plenty of individual ontologies has emerged recently, some of them representing overlapping contents [10]. Thus, the fact of using different ontologies increases heterogeneity problems to a different level.

Semantic alignment between ontologies is a solution to the heterogeneity problem. It can be considered the result of the ontology matching process which deals with finding the correspondences between semantically related entities of different ontologies [8]. The existence of a semantic alignment between ontologies is a necessary precondition to establish interoperability between the involved entities using different individual ontologies. Moreover, human users want to access the knowledge represented in numerous different ontologies in order to ease the task of searching or browsing. In addition, new knowledge can be inferred by combining the information contained in the various ontologies [29]. Thus, ontology alignment is a crucial issue to resolve in any application involving more than one entity or party where semantic heterogeneity is an intrinsic problem [33].

Once semantic alignments are established between individual ontologies, a network of linked ontologies is created, whose dynamics can be captured by the alignment composition operation [12, 37]. Indeed, if we have an alignment between ontologies O_1 and O_2 and an alignment between ontologies O_2 and O_3, we can compose them and obtain an alignment between ontologies O_1 and O_3, thus depicting relations holding between the entities of O_1 and O_3 ontologies. This is essential in order to retain the consistency of a network of aligned ontologies in spite of changes in ontologies participating in the specific network. More precisely, as depicted in Figure 1, whenever an autonomous entity represented by an ontology joins an already established network of already aligned ontologies, it suffices to align it to a single anchor ontology, already participating in the network. The anchor alignment produced, is then composed to already established alignments involving the anchor ontology, producing a batch of composition-generated alignments that remain, even if later on the anchor ontology leaves the network [21].

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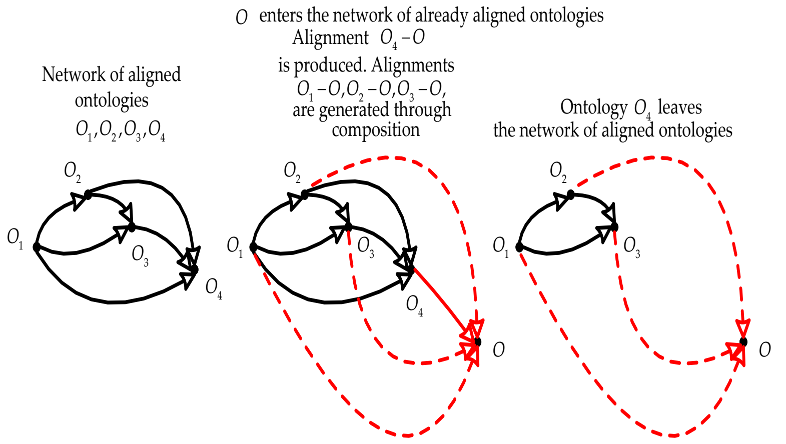

Fig 1: The significance of the composition of alignments in a network of aligned ontologies

At this point, a crucial issue is to provide rigorous methodological guidelines for networks of aligned ontologies engineering, in order to provide consistent such networks and thus be able to extract meaningful results.

Once two ontologies have been aligned, an elementary network of aligned ontologies has been created and which can then be populated with more ontologies. This network of aligned ontologies is a means of sharing and reuse. Sharing refers to the fact that many networks of aligned ontologies may use the same ontology for serving different purposes. Reuse means to build a new more complex network of aligned ontologies, by assembling already built elementary networks of aligned ontologies. The mechanism to obtain this is the composition of alignments.

Moreover, in networks of aligned ontologies, local entailments of standalone ontologies, induce relations between entities belonging to remote ontologies, via paths of consecutive ontologies and alignments, or provide new relations between local ontology entities, through path loops [14]. Indeed, if we have an alignment A_{12} between ontologies O_1 and O_2, an alignment A_{23} between ontologies O_2 and O_3, and an alignment A_{13} between ontologies O_1 and O_3, obtained by an alignment composition operator, new relations can be deduced, either relating ontological entities belonging to remote ontologies through a particular path of those ontologies and alignments, or relating ontological entities belonging to the same ontology, but through this particular path forming a loop starting and ending at a specific ontology. More precisely, as depicted in Figure 2, through the path $O_3 - A_{31} - O_1 - A_{12} - O_2 - A_{23} - O_3$, the relation $d_3 \le a_3$ results, while simultaneously $d_3 \perp a_3$ is revealed.

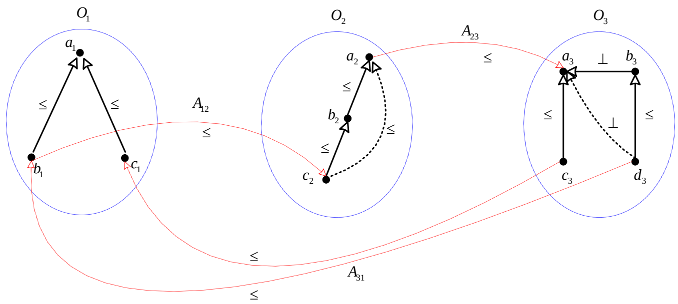

Fig 2: An example of a network of aligned ontologies

An this point, a crucial issue is to characterize the global, network induced entailments and propagate local knowledge through the network, in order to retain the consistency of the whole network and thus to extract meaningful results from the global knowledge emerged from a particular network of aligned ontologies.

Although the issues related to the research field of networks of aligned ontologies involve many discrete dimensions, such as algebra of relations [34], matching algorithms [32], ontology alignment techniques [8], etc., in this paper, we focus on offering suggestions for future research directions in using more elaborate categorical ideas, such as the Yoneda embedding functor and enriched categories, since we conjecture that significant improvements can be obtained only by addressing the important challenge of formalizing the basic building blocks of networks of aligned ontologies and the propagation of knowledge within them, in a way independent from their language representation and implementation.

3. CATEGORY THEORY IN ONTOLOGY OPERATIONS

In this section, some of the basic categorical constructions (see [2]) that are already used heavily in the formalizing ontologies and their operations, such as merging, alignment and mapping, are discussed. The section mainly constitutes a repetition of related work

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presented in [31], where, through Category Theory, the importance of an ontology alignment composition operator is underlined, in order to formally capture changes occurring in networks of aligned ontologies. The approach in this paper differs from our previous work in two main aspects. Firstly, we consider a different theoretical representation of ontologies, based on the path category of the quiver representing the ontology, in order to guarantee the composition of relations and define propagation matrices for propagating relations in a network of aligned ontologies, and secondly, here we are interested in characterizing entailments in networks of aligned ontologies.

Since Category Theory offers several ways in order to combine and integrate objects, it has been used as a mechanism to formalize ontology matching, providing operations to compose and decompose ontologies (alignment, merging, integration, mapping) [5, 17, 19, 38]. Since the basic objects in category theory are the relationships between ontological specifications and not the internal structure of a single knowledge representation, it permits to view global operations involving ontologies (like alignment, merging, matching), independently from the languages used to express their local entities and from the techniques used. In this section, the main results of this formalization are reviewed.
An ontology is a structure composed by concepts which are structured in a taxonomy (hierarchy of concepts) and relations, which are non taxonomic connections between the concepts.

In order to have a categorical view of ontologies, the category Ont of ontologies is defined in, for example, [5], where ontologies are considered as category objects, and pairs of functions (f,g) between a domain and a codomain ontology, are considered as the morphisms between objects, where f (and g), map concepts (respectively relations) of the domain ontology to concepts (respectively relations) of the codomain ontology. The morphisms are such that they preserve any hierarchy of concepts and any relations defined in the domain ontology, that is, if c_1 is a sub concept of c_2 in the domain ontology, $f(c_1)$ is a sub concept of $f(c_2)$ in the codomain ontology and if c_1 and c_2 are connected by the relation r in the domain ontology, $f(c_1)$ and $f(c_2)$ are connected by the relation $g(r)$ in the codomain ontology.

A category can be pictured as a graph (or diagram), where arrows are connecting the category objects.

The alignment between two ontologies O_1 and O_2, is the task of establishing binary relations between the entities of the two ontologies. Each binary relation can be decomposed into a pair of mappings from a common intermediate source ontology, O, [19]. The mappings from O to O_1 and from O to O_2, specify how the concepts and relations of O are understood in O_1 and O_2, respectively. This structure, comprising the ontologies O_1, O_2 and O, and the morphisms (f_1,g_1), (f_2,g_2), is called (due to its shape) a $V-$alignment (Figure 3) and is also called a span, in the Category Theory terminology.

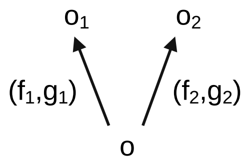

Fig 3: $V-$alignment

The operation of integrating two aligned ontologies into a single one, is called merging and can be accomplished with $V-$alignments. The ontology resulting from the unification process of merging, embodies the semantic differences of the two ontologies and collapses the semantic intersection between them. Merging of aligned ontologies can be described, in the Category Theory formalization, in terms of a Category Theory construct, called pushout. The pushout is a particular case of another construct, called colimit. The colimit, in the general case, entangles many objects, while the pushout entangles only three objects (the three categories O_1, O_2 and O in this case) and the two morphisms of the alignment diagram. The pushout is a new object O' (an ontology in this case), together with morphisms $(f_1,g_1)'$, $(f_2,g_2)'$, such that

$$(f_1,g_1)' \circ (f_1 g_1) = (f_2,g_2)' \circ (f_2,g_2) \qquad (3.1)$$

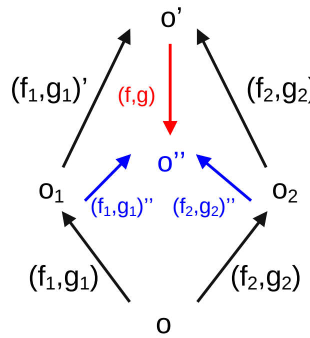

Fig 4: Merging through the pushout construct

The commutativity of the pushout diagram, means that components of O_1 and O_2 that are images of the same component in O (that is, the semantic intersection of O_1 land O_2), are collapsed in the resulting ontology O' (mapped to the same entity). But, this is exactly the definition of the merging operation! That is,

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the pushout ontology realizes the merging of O_1 and O_2 (Figure 4). Moreover, for any other object (ontology) O'' for which the commutativity holds, i.e. for which

$$(f_1,g_1)''\circ(f_1,g_1)=(f_2,g_2)''\circ(f_2,g_2) \qquad (3.2)$$

there exists a unique morphism (f,g) such that

$$(f,g)\circ(f_1,g_1)'=(f_1,g_1)'' \qquad (3.3)$$

and

$$(f,g)\circ(f_2,g_2)'=(f_2,g_2)'', \qquad (3.4)$$

that is, the pushout O' is the most compact ontology that can embody the union of O_1, O_2 which possibly comprises collapsed components (i.e., embodies the semantic differences and collapses the semantic intersection).

In Category Theory, dual concepts arise by the process of reversing all the morphisms in a diagram. Thus, the dual concept of pushout is a construct called pullback, which is a particular case of another construct called limit (dual of colimit). The pullback is used in order to formalize the matching operation [1], by which similarities between ontologies are detected. We start with what is called a $\Lambda-$alignment (Figure 5).

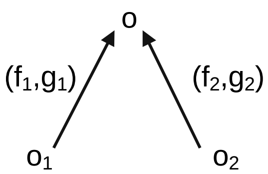

Fig 5: $\Lambda-$alignment

Here, O_1 and O_2 are the ontologies to be matched and O is an intermediate ontology that guides the matching. The pullback is a new ontology O', together with morphisms $(f_1,g_1)'$, $(f_2,g_2)'$, such that

$$(f_1,g_1)\circ(f_1,g_1)'=(f_2,g_2)\circ(f_2,g_2)', \qquad (3.5)$$

that is, the pullback O' embodies all information of O_1 and O_2 that is semantically equivalent.

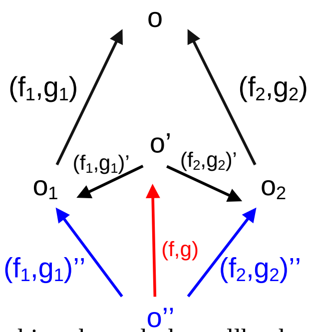

Fig 6: Matching through the pullback construct

The commutativity of the pullback diagram, means that components of O_1 and O_2 that have the same image in O (are semantically equivalent), are images of the same component in O'. But, this is exactly the definition of the matching operation! Thus, the pullback operation realizes the matching of O_1 and O_2 (Figure 6). Moreover, for any other object (ontology) O'' for which the commutativity holds, i.e.

$$(f_1,g_1)\circ(f_1,g_1)''=(f_2,g_2)\circ(f_2,g_2)'' \qquad (3.6)$$

there exists a unique morphism (f,g), such that

$$(f_1,g_1)'\circ(f,g)=(f_1,g_1)'' \qquad (3.7)$$

and

$$(f_2,g_2)'\circ(f,g)=(f_2,g_2)'', \qquad (3.8)$$

that is, the pullback O' is the biggest ontology that includes all the semantic intersection of O_1 land O_2.

Likewise, other operations involving manipulation of different alignments, as alignment composition, intersection and union, can be formulated in the categorical framework [38]. If we have alignments between ontologies O_1 and O_2, and between ontologies O_2 and O_3, we can compose them and obtain an alignment between ontologies O_1 and O_3, in the same way as composing spans in category theory, through the use of the pullback construct (Figure 7).

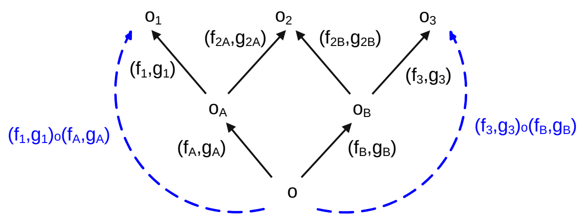

Fig 7: Composition of alignments

The ontology O, together with the morphisms $(f_1,g_1)\circ(f_A,g_A)$ and $(f_3,g_3)\circ(f_B,g_B)$ constitute the composition sought, where O, together with the morphisms (f_A,g_A), (f_B,g_B) is the pullback of the $\Lambda-$alignment of O_2, (O_2 with (f_{2A},g_{2A}), (f_{2B},g_{2B})).

In an analogous manner, the intersection between two alignments, (which depicts the mutually agreed correspondences of the two alignments), is formalized by the use of a limit, while the union of two alignments, (which gathers all the asserted relations specified in the two alignments), is formalized as the pushout of the intersection of the two alignments [38].

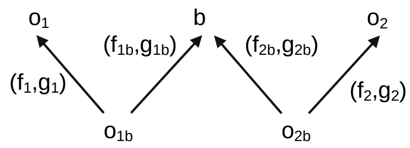

Fig 8: $W-$alignment

In cases where subsumption between the concepts of two ontologies is to be expressed, since this relation cannot be represented with the vocabulary of any of the two ontologies, it is externalized in an additional new ontology (called bridge ontology), as a bridge axiom. The diagram (Figure 8) depicts the situation, with the original ontologies O_1 and O_2 containing the concepts related via subsumption and the bridge ontology b containing the bridge axioms. The fact that there exist concepts of the ontologies O_1 and O_2 occurring within the bridge ontology, is represented by the two $V-$alignments between the bridge ontology and the ontologies O_1 and O_2 . Thus, what is called a $W-$alignment, is defined.

The merging operation in this case, is defined as the colimit of the alignment diagram and is computed by successive pushouts [38] (Figure 9).

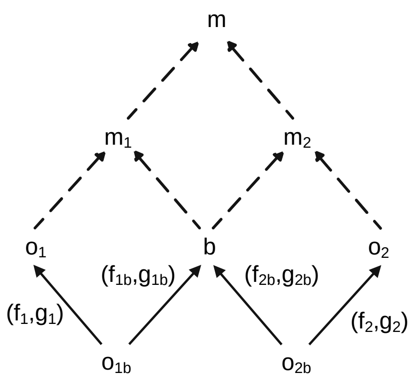

Fig 9: Merging with $W-$alignments

In a similar way, one can compose $W-$alignments. If a $W-$alignment exists between ontologies O_1 and O_2 with bridge ontology b_1 and if also a $W-$alignment exists between ontologies O_2 and O_3 with bridge ontology b_2, by composing the two $W-$alignments, it results that a $W-$alignment exists between ontologies O_1 and O_3, with bridge ontology b, which is obtained if the merging operation is applied to the bridge ontologies b_1 and b_2. The problem of this approach, consists in incorporating in the new bridge ontology b bridge axioms from the ontologies b_1, b_2 and O_2, that might be irrelative to O_1 and O_3.

Another solution to the problem of more elaborate relationships (subsumption, strict inclusion, strict containment, disjointness, overlapping with partial disjointness, temporal relations), between ontology entities, is to enhance the category of ontologies with more elaborate morphisms that denote the relationship that holds between the syntactic entities of the two ontologies (subsumption, strict inclusion etc.) [38]. In this case, when applying the composition operation, if an entity in ontology O_2 has an elaborate relation to entities in the ontologies O_1 and O_3, there is some kind of relation between the two entities in O_1 and O_3. The latter relation depends strongly on the former one. For example, if an entity in O_1 is related to an entity in O_2 with strict inclusion and the same entity in O_2 is related to an entity in O_3 with strict containment, then the entity in O_1 can be related to the entity in O_3 by either of the following relationships: equivalence, strict inclusion, strict containment, disjointness, overlapping with partial disjointness.

4. RELATED THEORETICAL WORK IN NETWORKS OF ONTOLOGIES

This section reports on some recent work in networks of ontologies linked by alignments. When dealing with a network of ontologies related by alignments, the main issue to consider is: given the semantics of the constituent ontologies and/or alignments, what are the consequences entailed in the whole network. In this perspective, two main lines of research have been followed: designing a theoretical framework to formally define composition of ontology alignments and checking the semantic consistency of a whole network of aligned ontologies. In [37], three different semantics for distributed systems, defined as sets of ontologies interconnected by alignments, have been introduced, upon which a mapping composition operator has been defined. The authors distinguish between syntactic composition and semantic composition, which requires semantic consistency to be preserved when indirectly linking two ontologies. In [9], an algebra for ontology alignment relations has been introduced with some considerations on how mapping composition can be performed. In [39], a formalism, IDDL, which treats local knowledge (ontologies) and global knowledge (alignments) separately by distinguishing local interpretations and global interpretation has been proposed and an algorithm for consistency checking has been presented. Authors' main contribution is a reasoning procedure which determines whether or not an IDDL distributed system is consistent. In [24], network of ontologies and alignments have been investigated by introducing the notion of hyper ontology, which serves to study (in)consistency propagation in connected alignments. In [12], two notions of consequences called a-consequence and ω-consequence have been introduced in order to define a consistent network of ontologies. In [14], a formal model to represent networks of ontologies and alignments has been provided, by introducing the notion of Semantic Flow Network, with the aim to investigate the problem of composite mapping discovery.

5. ONTOLOGY REPRESENTATION BY A PATH CATEGORY

In networks of aligned ontologies, local entailments of standalone ontologies, induce relations between entities belonging to remote ontologies, via paths of consecutive ontologies and alignments, or provide new relations between local ontology entities, through path loops. By proposing a formalism, based on Category Theory and contra variant represent able functors, we intend to characterize these global, network induced entailments and propagate local knowledge through the network, by using matrix-like operators exploiting composition defined over an algebra of binary relations.
In this paper, we focus only on the first step of this attempt, which is the ontology representation as a path category upon which the propagation matrix-based operations will be defined, in conjunction with the contra variant represent able functors.

A directed graph, or digraph, is characterized by a set of nodes (or vertices), connected by edges (or arcs, or arrows) with an associated direction. A quiver, or multigraph is a directed graph where, moreover, loops (i.e. edges with the same source and target vertices) as well as multiple edges between two vertices, are allowed. Thus, in simple digraphs, the arcs constitute a set of ordered pairs, while in quivers they constitute a multiset (a set of sets) of ordered pairs. A labeled quiver is a multigraph with labeled vertices and edges, the vertex and edge labels taking values from finite alphabets. Formally, a *labeled quiver* is an 8-tuple $Q_L = (\Sigma_V, \Sigma_E, V, E, s, t, l_V, l_E)$, where

- V is the set of vertices with $l_V : V \to \Sigma_V$ a map assigning to each vertex a label from the finite alphabet Σ_V
- E is the set of edges with $l_E : E \to \Sigma_E$ a map assigning to each edge a label from the finite alphabet Σ_E
- $s : E \to V$ and $t : E \to V$ are two maps defining the source and target vertex of each edge.

In the above formal definition, we avoided the use of multisets, in order to characterize the edges, since we consider that edges with both the same source and target vertices are distinguished by different labels.

On the other side, since an ontology describes objects (instances) and classes (concepts), attributes that objects and classes can have and relationships by which instances and concepts are related to one another, it can be represented by a labeled quiver, with labeled vertices representing instances and concepts and labeled edges representing, either relations by which instances and concepts are connected, or attributes pointing from a concept, or instance to a set of admissible values for that attribute.

Since, moreover, in ontologies binary relations can be composed along paths, according to the rules of an algebra of binary relations, thus entailing new relations and since it is also primordial to check the equivalence of relations emerging from compositions along different paths connecting the same source and target entities, and both these two characteristics distinguish categories from graphs, we firstly consider the *path category* of the quiver representing the ontology.

The path category, or free category, associated with a quiver, is the category having as objects the vertices of the quiver and having as morphisms from vertex V_0 to vertex V_n, lists of the form $(V_n, e_n, V_{n-1}, e_{n-1}, V_{n-2}, ..., V_2, e_2, V_1, e_1, V_0)$, with $e_1 : V_0 \to V_1$, $e_2 : V_1 \to V_2$, …, $e_{n-1} : V_{n-2} \to V_{n-1}$, $e_n : V_{n-1} \to V_n$ being consecutive edges of the quiver, forming a path from V_0 to V_n and labeled with $e_1, e_2, ..., e_{n-1}, e_n$, respectively. We informally write $e_1, e_2, ..., e_{n-1}, e_n$ for the respective morphisms (V_1, e_1, V_0), (V_2, e_2, V_1), …, $(V_{n-1}, e_{n-1}, V_{n-2})$, (V_n, e_n, V_{n-1}).

The composition of morphisms in the path category becomes the concatenation of consecutive paths.
Finally, we generate the category representing the ontology. This category has the same objects as those at the path category and in the place of the morphism $(V_n, e_n, V_{n-1}, e_{n-1}, V_{n-2}, ..., V_2, e_2, V_1, e_1, V_0)$ of the path category, it has a morphism labeled by the composition of relations $e_n \circ e_{n-1} \circ ... \circ e_2 \circ e_1$. The composition of morphisms becomes here the composition of binary relations in the respective algebra of binary relations. We supplement the category with identity morphisms being the unity element of composition in the algebra of binary relations. The associatively of the law of composition of relations, guarantees by construction the associatively of the operation of composition of morphisms in the category.

Thus, by representing an ontology through a path category, we are able to check the consistency of relations emerging from compositions along different paths and the entailment of new relations.

6. CONCLUSIONS AND FUTURE WORK

In this paper, by representing an ontology as a path category, we provide future directions in characterizing entailments in a network of aligned ontologies. This comprises the first step of a theoretical framework suitable to detect inconsistencies, that is, conceptual errors in order to revise the knowledge emerging from the whole network and to reason on ontologies which are independently conceived and related by means of alignments.

Three main directions are considered here, in order to extend this work and, thus, to provide a categorical framework for characterizing entailments in networks of aligned ontologies, that is, the result of the interaction among their components (constituent ontologies and alignments: (1) the definition of an algebra of binary relations based on the algebra of relations of Tarski [34], in order to characterize the relations holding between entities, either two local entities of an individual ontology, or between entities belonging to separate remote ontologies, which are connected via alignments, (2) the representation of an ontology by means of a propagation matrix based on the idea of represent able contra variant functors, in order to provide a formulation for propagating relations in a network of ontologies and alignments, and (3) the definition of composition operators based on the categorical notion of natural transformation, in order to combine the propagation matrices along chains of ontologies and alignments.

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